# ON THE INVERSE MEIJER TRANSFORM OF THE G-FUNCTION

by

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1. A generalization of the classical LAPLACE transform

$$\phi(p) = p \int_0^\infty e^{-pt} h(t) \, dt \tag{1.1}$$

has been introduced by MEIJER [4] in the form

$$\phi(p) = \left(\frac{2}{\pi}\right)^{\frac{1}{2}} p \int_0^\infty (pt)^{\frac{1}{2}} K_\nu(pt) h(t) \, dt. \tag{1.2}$$

VARMA [10] has also given a generalization of (1.1) by the integral equation

$$\phi(p) = p \int_0^\infty e^{-\frac{1}{2}pt} (pt)^{m-\frac{1}{2}} W_{k,m}(pt) h(t) \, dt. \tag{1.3}$$

It is well known that (1.2) and (1.3) reduce to (1.1) when $\nu = \pm \frac{1}{2}$ and $k + m = \frac{1}{2}$ respectively, on account of the identities

$$K_{\pm\frac{1}{2}}(x) = \left(\frac{\pi}{2x}\right)^{\frac{1}{2}} e^{-x} \text{ and } W_{\frac{1}{2}-m, m}(x) = e^{-\frac{1}{2}x} x^{\frac{1}{2}-m} \tag{1.4}$$

As usual, in (1.2) $\phi(p)$ will be called the MEIJER transform of $h(t)$ and $h(t)$ the inverse MEIJER transform of $\phi(p)$.

Throughout this note, the symbols $\phi(p) \doteqdot h(t)$, $\phi(p) \underset{\nu}{\overset{k}{\doteqdot}} h(t)$ and $\phi(p) \underset{k,m}{\overset{v}{=}} h(t)$ will be used to denote (1.1), (1.2) and (1.3) respectively.

The purpose of this note is to investigate the close connections existing between MEIJER and VARMA transforms. The results have been given in the form of two theorems. The inverse MEIJER trans-

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form of the $G$-function has been derived by the application of a sequence involving LAPLACE, MEIJER and VARMA transforms, which follows as a particular case of theorem 1.

The importance of the results obtained lies in the fact that they contain the MEIJER's $G$-function which is the generalization of nearly all the important special functions occurring in Applied Mathematics. As such, several curious formulae involving WHITTAKER functions, BESSEL functions, LEGENDRE functions, etc., their combinations and related functions can be deduced as special cases. Some formulae due to MEIJER, RAGAB and SAXENA follow as particular cases of our results.

The symbol $\Delta(l,\ a)$ has been used to represent the set of parameters

$$\frac{a}{l},\ \frac{a+1}{l},\ \ldots\ \frac{a+l-1}{l}$$

In what follows $n$ and $s$ are positive integers.

2. THEOREM 1.

If

$$\phi\,(p) \underset{v}{\overset{k}{=\!=}} t^{l}\,f(t)$$

and

$$\psi\,(p) \underset{k,\ m}{\overset{v}{=\!=}} t^{\frac{n}{2s}(\lambda+\frac{3}{2})}\,f\,(t^{\frac{n}{2s}}) \tag{2.1}$$

then

$$\phi\,(p) = p^{\lambda+\frac{3}{2}-l-\frac{2s}{n}}\,(2s)^{l+\frac{2s}{n}-\lambda-1}\,(2\pi)^{\frac{n}{2}-s}\,n^{-(k+m)}$$

$$\times\int_{\xi}^{\infty}\psi(t)\,G^{2s+n,\ 0}_{2n,\ 2s+n}\left[\left(\frac{p}{2s}\right)^{2s}\left(\frac{n}{t}\right)^{n}\middle|\begin{matrix}\Delta(n,\ 2),\ \Delta(n,\ 2m+2)\\ \Delta(s,\frac{e\pm v-\lambda}{2}+\frac{s}{n}),\ \Delta(n,\frac{5+2m-2k}{2})\end{matrix}\right]dt \tag{2.2}$$

where $\xi = 0$ if $\frac{n}{2s} < 1$ and $\xi = p$ as $\frac{n}{2s} \to 1$ provided that the integral is convergent, the MEIJER transform of $|\ t^{l}\,f\,(t)\ |$ and the VARMA transform of $|\ t^{\frac{n}{2s}(\lambda+\frac{3}{2})}\,f\,(t^{\frac{n}{2s}})\ |$ exist and $R\,(p) > 0$.

PROOF. We know that [8]

$$(p^{\frac{n}{2s}})^{l-\lambda} K_\nu (a p^{\frac{n}{2s}}) \underset{k,\,m}{\overset{v}{=\!=}} a^{\lambda-l}\, 2^{l-\lambda-1}\, (2\pi)^{\frac{n+1-2s}{2}}\, s^{l-\lambda}\, n^{-(k+m)}$$

$$\times\, G^{\,2s+n,\,0}_{\,2n,\,2s+n}\left[\left(\frac{a}{2s}\right)^{2s}\left(\frac{n}{t}\right)^{n}\,\middle|\, \begin{matrix}\Delta(n,\,1),\ \Delta(n,\,2m+1)\\ \Delta\left(s,\dfrac{l\pm\nu-\lambda}{2}\right),\ \Delta\left(n,\dfrac{2m+3-2k}{2}\right)\end{matrix}\right] \tag{2.3}$$

where

$$|\arg a| < \left(1-\frac{n}{2s}\right)\frac{\pi}{2} \text{ and } R(p) > 0 \text{ if } \frac{n}{2s} < 1 \text{ and } t \geq a \text{ as } \frac{n}{2s} \to 1.$$

Applying the PARSEVAL-GOLDSTEIN theorem for the VARMA transform to (2.1) and (2.3) we find that

$$\int_0^\infty t^{\frac{n}{2s}(l+\frac{3}{2})-1} f(t^{\frac{n}{2s}})\, K_\nu(a t^{\frac{n}{2s}})\, dt = a^{\lambda-l}\, 2^{l-\lambda-1}\, (2\pi)^{\frac{n+1-2s}{2}}\, s^{l-\lambda}\, n^{-(k+m)}$$

$$\times \int_\eta^\infty \frac{\psi(t)}{t}\, G^{\,2s+n,\,0}_{\,2n,\,2s+n}\left[\left(\frac{a}{2s}\right)^{2s}\left(\frac{n}{t}\right)^{n}\,\middle|\, \begin{matrix}\Delta(n,\,1),\ \Delta(n,\,2m+1)\\ \Delta\left(s,\dfrac{l\pm\nu-\lambda}{2}\right),\ \Delta\left(n,\dfrac{2m+3-2k}{2}\right)\end{matrix}\right] dt$$

where $\eta = 0$ if $\frac{n}{2s} < 1$ and $\eta = a$, as $\frac{n}{2s} \to 1$.

On interpreting the left hand side of the above equation from (1.2) we arrive at the result.

A special case of the theorem for $n \to 2$, $s \to 1$ and $k + m = \frac{1}{2}$ has been given by SHARMA [9, $p$ 112].

COROLLARY. If we take $n = s = 1$, $\lambda = -\frac{3}{2}$, $l = \mu - \frac{1}{2}$ and $k + m = \frac{1}{2}$ in (2.2), the $G$-function in the integrand equals WHITTAKER function and it is seen that if

$$\psi(p) \doteqdot f(\sqrt{t})$$

and

$$\phi(p) \underset{\nu}{\overset{k}{=\!=}} t^{\mu-\frac{1}{2}} f(t)$$

then

$$\phi(p) = \left(\frac{2}{p}\right)^{\mu-\frac{1}{2}} \frac{1}{\sqrt{\pi}} \int_0^\infty t^{\frac{\mu}{2}-1} e^{-\frac{1}{2}t}\, W_{\frac{\mu}{2},\frac{\nu}{2}}(t)\, \psi\left(\frac{p^2}{4t}\right) dt \tag{2.4}$$

provided that the integral is convergent, the LAPLACE transform of $|f(\sqrt{t})|$, the MEIJER transform of $|t^{\mu-\frac{1}{2}} f(t)|$ exist, and $R(p) > 0$.

This can also be enunciated in the following interesting form.

If

$$\psi(p) \doteqdot f(\sqrt{t})$$

and

$$\phi(p) \underset{\nu}{\overset{k}{=\!=}} t^{\mu-\frac{1}{2}} f(t) \tag{2.5}$$

then

$$\sqrt{\pi}\, p^{\frac{\nu}{2}+\frac{1}{4}} \phi(2\sqrt{p}) \underset{\frac{\mu}{2},\,\frac{\nu}{2}}{\overset{v}{=\!=}} t^{\frac{\mu-\nu-1}{2}} \psi\left(\frac{1}{t}\right) \tag{2.6}$$

provided that the LAPLACE, MEIJER and VARMA transforms of $|f(\sqrt{t})|$, $|t^{\mu-\frac{1}{2}} f(t)|$ and $\left| t^{\frac{\mu-\nu-1}{2}} \psi\left(\frac{1}{t}\right) \right|$ respectively exist and $R(p) > 0$.

A particular case of this has been reproduced in [2, *p.* 19].

3. *Applications of the corollary.*

If we take [8]

$$\sqrt{\pi}\, p^{\frac{\nu}{2}+\frac{1}{4}} \phi(2\sqrt{p}) = p^{\varrho} G^{\alpha,\,\beta}_{\gamma,\,\delta}\left( a p^{-\frac{n}{s}} \,\middle|\, \begin{matrix} a_1, \ldots, a_\gamma \\ b_1, \ldots, b_\delta \end{matrix} \right)$$

$$\underset{\frac{\mu}{2},\,\frac{\nu}{2}}{\overset{v}{=\!=}} t^{-\varrho} (2\pi)^{\frac{1}{2}(n-1)+(1-s)\left(\alpha+\beta-\frac{\gamma}{2}-\frac{\delta}{2}\right)} n^{\varrho-\frac{\mu}{2}-\frac{\nu}{2}} s^{\sum_{i=1}^{\delta} b_i - \sum_{\nu=1}^{\gamma} a_\nu + \frac{1}{2}\gamma - \frac{1}{2}\delta + 1}$$

$$\times\, G^{s\alpha,\, s\beta+n}_{s\gamma+n,\, s\delta+2n}\left[\frac{a^s t^n}{s^{s(\delta-\gamma)} n^n} \,\middle|\, \begin{matrix} \Delta(s, a_1), \ldots, \Delta(s, a_\gamma), \Delta\left(n, \frac{\mu+2\varrho-\nu-1}{2}\right) \\ \Delta(s, b_1), \ldots, \Delta(s, b_\delta), \Delta(n, \varrho), \Delta(n, \varrho-\nu) \end{matrix} \right] \tag{3.1}$$

$$= t^{\frac{\mu-\nu-1}{2}} \psi\left(\frac{1}{t}\right),$$

where

$$R\,(\min\ n\, b_h + \tfrac{\nu}{2}\, s \pm \tfrac{\nu}{2}\, s + s - s\varrho) > 0, \text{ for } h = 1, 2, \ldots \alpha$$

$$\alpha + \beta > \tfrac{1}{2}\,\gamma + \tfrac{1}{2}\,\delta + \tfrac{n}{2s},\ |\arg a| < (\alpha + \beta - \tfrac{1}{2}\,\gamma - \tfrac{1}{2}\,\delta - \tfrac{n}{2s})\,\pi,$$

and $R\,(p) > 0$.

then [7, $p$ 40], we have

$$\psi(p) = p^{\frac{1}{2}(\mu+2\varrho-\nu-1)}\,(2\pi)^{\frac{1}{2}(n-1)+(1-s)(\alpha+\beta-\frac{\gamma}{2}-\frac{\delta}{2})}\,n^{\varrho-\frac{\mu}{2}-\frac{\nu}{2}}\,s^{\sum_{i=1}^{\delta} bi-\sum_{\nu=1}^{\gamma} a_\nu+\frac{1}{2}\gamma-\frac{1}{2}\delta+1}$$

$$\times\, G^{s\alpha,\, s\beta+n}_{s\gamma+n,\, s\delta+2n}\left[\frac{a^s\, s^{s(\gamma-\delta)}}{n^n\, p^n}\,\middle|\,\begin{matrix}\Delta(s,a_1),\ldots\Delta(s,a_\gamma),\Delta\left(n,\dfrac{\mu+2\varrho-\nu-1}{2}\right)\\ \Delta(s,b_1),\ldots\Delta(s,b_\delta),\Delta(n,\varrho),\Delta(n,\varrho-\nu)\end{matrix}\right]$$

$$\doteqdot n^{2\varrho-\nu-1}\, s^{\sum_{i=1}^{\delta} bi-\sum_{\nu=1}^{\gamma} a_\nu+\frac{1}{2}\gamma-\frac{1}{2}\delta+1}\,(2\pi)^{(n-1)+(1-s)(\alpha+\beta-\frac{\gamma}{2}-\frac{\delta}{2})}$$

$$\times\, t^{\frac{1}{2}(\nu+1-\mu-2\varrho)}\, G^{s\alpha,\, s\beta}_{s\gamma,\, s\delta+2n}\left[\frac{t^n\, a^s}{s^{s(\delta-\gamma)}\, n^{2n}}\,\middle|\,\begin{matrix}\Delta(s,a_1),\ldots\Delta(s,a_\gamma)\\ \Delta(s,b_1),\ldots\Delta(s,b_\delta),\Delta(n,\varrho),\Delta(n,\varrho-\nu)\end{matrix}\right] = f(\sqrt{t}) \tag{3.2}$$

where

$$R\,\{\min\ 2nb_h + s\,(\nu + 3 - \mu - 2\varrho)\} > 0, \text{ for } h = 1, 2, \ldots \alpha$$

$$(\alpha + \beta) > \tfrac{1}{2}\,\gamma + \tfrac{1}{2}\,\delta + \tfrac{n}{s},\ |\arg a| < (\alpha + \beta - \tfrac{1}{2}\,\gamma - \tfrac{1}{2}\,\delta - \tfrac{n}{s})\,\pi$$

and $R\,(p) > 0$.

From (2.5) it is seen that

$$p^\sigma\, G^{\alpha,\,\beta}_{\gamma,\,\delta}\left[a\, p^{-\frac{2n}{s}}\,\middle|\,\begin{matrix}a_1, \ldots a_\gamma\\ b_1, \ldots b_\delta\end{matrix}\right]$$

$$\overset{k}{\underset{\nu}{=}} \sqrt{2\pi}\,(2n)^{\sigma-\frac{1}{2}}\,(2\pi)^{(n-1)+(1-s)(\alpha+\beta-\frac{\gamma}{2}-\frac{\delta}{2})}\, s^{\sum_{i=1}^{\delta} bi-\sum_{\nu=1}^{\gamma} a_\nu+\frac{1}{2}\gamma-\frac{1}{2}\delta+1}$$

$$\times\, t^{-\sigma}\, G^{s\alpha,\, s\beta}_{s\gamma,\, s\delta+2n}\left[\frac{a^s\, t^{2n}}{s^{s(\delta-\gamma)}\,(2n)^{2n}}\,\middle|\,\begin{matrix}\Delta(s,a_1),\ldots,\Delta(s,a_\gamma)\\ \Delta(s,b_1),\ldots\Delta(s,b_\delta),\Delta\left(n,\dfrac{2^\sigma \pm 2\nu + 1)}{4}\right)\end{matrix}\right] \tag{3.3}$$

where

$$R\,(\min\ 2nb_h \pm \nu s + \tfrac{3}{2}\ s - s\sigma) > 0, \text{ for } h = 1, 2, \ldots, \alpha\,;$$

$\alpha + \beta > \frac{1}{2}\gamma + \frac{1}{2}\delta + \frac{n}{s}$, $|\arg\ a| < (\alpha + \beta - \frac{1}{2}\gamma - \frac{1}{2}\delta - \frac{n}{s})\,\pi$ and $R\,(p) > 0$.

On specializing the parameters in (3.3) and using the identities involving the $G$-functions [1, pp. 209, eqn. 9, 219 eqn. 47, 221 eqns. 66, 68, 69 and 222 eqn. 73], the following results can be easily deduced. Some of these results also supply new integral representations for the Bessel functions.

$$2\,p^{\sigma}\ K_{\lambda}\,(a\,p^{-\frac{n}{s}}) \underset{\nu}{\overset{k}{=\!=}} \sqrt{2\pi}\ (2n)^{\sigma-\frac{1}{2}}\ (2\pi)^{n-s}$$

$$\times\ t^{-\sigma}\ G^{2s,\,0}_{0,\,2s+2n}\left[\left(\frac{a}{2s}\right)^{2s}\left(\frac{t}{2n}\right)^{2n}\,\middle|\,\Delta(s, \pm\tfrac{\lambda}{2}), \Delta(n, \frac{2\sigma \pm 2\nu + 1}{4})\right] \tag{3.4}$$

where $R\,(\frac{3}{2}\ s \pm n\,\lambda \pm \nu s - \sigma s) > 0$, $s > n$, $|\arg\ a^{2s}| < (s - n)\,\pi$ and $R\,(p) > 0$. When $n = s$, $a$ should be real and positive.

When $n = s = 1$ (3.4) reduces to a result due to Meijer [3, p. 352].

$$2p^{\sigma}\ K_{\lambda}\,(ap^{\frac{n}{s}}) \underset{\nu}{\overset{k}{=\!=}} \sqrt{2\pi}\ (2n)^{\sigma-\frac{1}{2}}\ (2\pi)^{n-s}$$

$$\times\ t^{-\sigma}\ G^{2s,\,0}_{2n,\,2s}\left[\left(\frac{a}{2s}\right)^{2s}\left(\frac{2n}{t}\right)^{2n}\,\middle|\,\begin{matrix}\Delta\,(n, \frac{\pm 2\nu - 2\sigma + 3}{4})\\ \Delta\,(s, \pm\,\frac{\lambda}{2})\end{matrix}\right] \tag{3.5}$$

where

$$|\arg\ a^{2s}| < (s - n)\,\pi,\ s > n \text{ and } R\,(p) > 0.$$

$$p^{\sigma}\ e^{-\frac{1}{2}\,ap^{-\frac{2n}{s}}}\ W_{k,\,m}\,(ap^{-\frac{2n}{s}}) \underset{\nu}{\overset{k}{=\!=}} (2n)^{\sigma-\frac{1}{2}}\ (2\pi)^{\frac{1}{2}(2n-s)}\ s^{k+\frac{1}{2}}$$

$$\times\ t^{-\sigma}\ G^{2s,\,0}_{s,\,2s+2n}\left[\left(\frac{a}{s}\right)^{s}\left(\frac{t}{2n}\right)^{2n}\,\middle|\,\begin{matrix}\Delta\,(s, 1 - k)\\ \Delta(s, \frac{1}{2} \pm m), \Delta(n, \frac{2\sigma \pm 2\nu + 1}{4})\end{matrix}\right] \tag{3.6}$$

where $R\,(n + \frac{3}{2}\ s \pm 2m\,n \pm \nu s - \sigma s) > 0$, $|\arg\ a^{s}| < (s - 2n)\frac{\pi}{2}$ $R\,(p) > 0$ and $s > 2n$.

$$p^{\sigma}\, e^{-\frac{1}{2}ap^{\frac{2n}{s}}}\, W_{k,m}\,(ap^{\frac{2n}{s}}) \overset{k}{\underset{\nu}{=}} (2n)^{\sigma-\frac{1}{2}}\, (2\pi)^{\frac{1}{2}(2n-s)}\, s^{k+\frac{1}{2}}$$

$$\times\, t^{-\sigma}\, G^{2s,\,0}_{s+2n,\,2s}\left[\left(\frac{a}{s}\right)^{s}\left(\frac{2n}{t}\right)^{2n}\middle|\begin{matrix}\Delta(s,\,1-k),\,\Delta\!\left(n,\,\pm\dfrac{2\nu-2\sigma+3}{4}\right)\\ \Delta(s,\,\frac{1}{2}\pm m)\end{matrix}\right] \tag{3.7}$$

where $|\arg a^{s}| < (s-2n)\frac{\pi}{2}$, $s > 2n$ and $R(p) > 0$.

$$p^{\sigma}\, e^{\frac{1}{2}ap^{-\frac{2n}{s}}}\, W_{k,m}\,(ap^{-\frac{2n}{s}}) \overset{k}{\underset{\nu}{=}} \frac{(2n)^{\sigma-\frac{1}{2}}\,(2\pi)^{n+1-\frac{3}{2}s}\, s^{\frac{1}{2}-k}}{\Gamma(\frac{1}{2}-k+m)\,\Gamma(\frac{1}{2}-k-m)}$$

$$\times\, t^{-\sigma}\, G^{2s,\,s}_{s,\,2s+2n}\left[\left(\frac{a}{s}\right)^{s}\left(\frac{t}{2n}\right)^{2n}\middle|\begin{matrix}\Delta(s,\,1+k)\\ \Delta(s,\,\frac{1}{2}\pm m),\,\Delta\!\left(n,\,\dfrac{2\sigma\pm2\nu+1}{4}\right)\end{matrix}\right] \tag{3.8}$$

where $R\,(n \pm 2mn \pm \nu s + \frac{3}{2}s - \sigma s) > 0$, $|\arg a^{s}| < (3s-2n)\frac{\pi}{2}$ $3s > 2n$ and $R(p) > 0$.

$$p^{\sigma}\, e^{\frac{1}{2}ap^{\frac{2n}{s}}}\, W_{k,m}\,(ap^{\frac{2n}{s}}) \overset{k}{\underset{\nu}{=}} \frac{(2n)^{\sigma-\frac{1}{2}}\,(2\pi)^{n+1-\frac{3}{2}s}\, s^{\frac{1}{2}-k}}{\Gamma(\frac{1}{2}-k+m)\,\Gamma(\frac{1}{2}-k-m)}$$

$$\times\, t^{-\sigma}\, G^{2s,\,s}_{s+2n,\,2s}\left[\left(\frac{a}{s}\right)^{s}\left(\frac{2n}{t}\right)^{2n}\middle|\begin{matrix}\Delta(s,\,1+k),\,\Delta\!\left(n,\,\pm\dfrac{2\nu-2\sigma+3}{4}\right)\\ \Delta(s,\,\frac{1}{2}\pm m)\end{matrix}\right] \tag{3.9}$$

where $|\arg a^{s}| < (3s-2n)\frac{\pi}{2}$, $3s > 2n$ and $R(p) > 0$.

$$p^{\sigma}\, K_{\lambda}\,(ap^{-\frac{n}{s}})\, K_{\mu}\,(ap^{-\frac{n}{s}}) \overset{k}{\underset{\nu}{=}} 2^{-1}\sqrt{\pi}\,(2n)^{\sigma-\frac{1}{2}}\,(2\pi)^{n+\frac{1}{2}-s}\, s^{-\frac{1}{2}}$$

$$\times\, t^{-\sigma}\, G^{4s,\,0}_{2s,\,4s+2n}\left[\left(\frac{a}{s}\right)^{2s}\left(\frac{t}{2n}\right)^{2n}\middle|\begin{matrix}\Delta(s,0),\,\Delta(s,\frac{1}{2})\\ \Delta(s,\pm\frac{1}{2}(\mu+\lambda)),\Delta(s,\pm\frac{1}{2}(\mu-\lambda)),\Delta\!\left(n,\dfrac{2\sigma\pm2\nu+1}{4}\right)\end{matrix}\right] \tag{3.10}$$

where

$$R\,[\pm n\,(\mu+\lambda) + \tfrac{3}{2}s - \sigma s \pm \nu s] > 0,\ s > n$$

$R\,[\pm n\,(\mu-\lambda) + \frac{3}{2}s - \sigma s \pm \nu s] > 0$, $|\arg a^{2s}| < (s-n)\,\pi$ and $R(p) > 0$.

$$p^{\sigma} W_{k,m}(ap^{\frac{n}{s}}) W_{-k,m}(ap^{\frac{n}{s}}) \overset{k}{\underset{v}{=\!=}} \pi^{-\frac{1}{2}} (2n)^{\sigma-\frac{1}{2}} (2\pi)^{n+\frac{1}{2}-s} s^{\frac{1}{2}}$$

$$\times t^{-\sigma} G^{4s,\,0}_{2s+2n,\,4s}\left[\left(\frac{a}{2s}\right)^{2s}\left(\frac{2n}{t}\right)^{2n} \middle| \begin{matrix} \Delta(s,\ 1 \pm k),\ \Delta\left(n,\ \frac{\pm 2v - 2\sigma + 3}{4}\right) \\ \Delta(s,\ \frac{1}{2}),\ \Delta(s,\ 1),\ \Delta(s,\ \frac{1}{2} \pm m) \end{matrix}\right] \quad (3.11)$$

where $|\arg a^s| < (s-n)\frac{\pi}{2}$, $s > n$ and $R(p) > 0$.

A particular case of (3.11) for $v = \pm\frac{1}{2}$ and $n = 1$ was given by RAGAB [5, p, 581].

4. THEOREM 2.

If

$$\phi(p) \overset{k}{\underset{v}{=\!=}} t^{\varrho} f(t)$$

and

$$\psi(p) \overset{v}{\underset{k,m}{=\!=}} t^{-\frac{n}{2s}(\lambda+\frac{3}{2})} f(t^{-\frac{n}{2s}})$$

then

$$\phi(p) = p^{\lambda+\frac{3}{2}+\frac{2s}{n}-\varrho} (2s)^{\varrho-\lambda-1-\frac{2s}{n}} (2\pi)^{\frac{1}{2}(n-2s)} (n)^{-(k+m)}$$

$$\times \int_0^\infty \psi(t).\, G^{2s,\,n}_{n,\,2s+2n}\left[\left(\frac{p}{2s}\right)^{2s}\left(\frac{t}{n}\right)^{n} \middle| \begin{matrix} \Delta\left(n, \frac{2k-2m-3}{2}\right) \\ \Delta\left(s, \frac{\varrho \pm v - \lambda}{2} - \frac{s}{n}\right), \Delta(n,\ -1), \Delta(n,\ -2m-1) \end{matrix}\right] dt$$

(4.1)

provided that the integral is convergent, the MEIJER transform of $|t^{\varrho} f(t)|$ and the VARMA transform of $|t^{-\frac{n}{2s}(\lambda+\frac{3}{2})} f(t^{-\frac{n}{2s}})|$ exist, $R(\varrho \pm v + 2s + 2ms \pm 2ms - \lambda) > 0$, $2s > n$ and $R(p) > 0$.

The proof is similar to that of theorem 1. The following result is required instead of (2.3).

$$(p^{-\frac{n}{2s}})^{\varrho-\lambda} K_\nu \left(a p^{-\frac{n}{2s}}\right) \underset{k,\, m}{\overset{v}{=\!=}} a^{\lambda-\varrho}\, 2^{\varrho-\lambda-1}\, (2\pi)^{\frac{1}{2}(n+1-2s)}\, s^{\varrho-\lambda}\, n^{-(k+m)}$$

$$\times G^{2s,\, n}_{n,\, 2s+2n}\left[\left(\frac{a}{2s}\right)^{2s}\left(\frac{t}{n}\right)^{n} \middle| \begin{matrix} \Delta\left(n, \dfrac{2k-2m-1}{2}\right) \\ \Delta\left(s, \dfrac{\varrho-\lambda\pm\nu}{2}\right), \Delta(n, 0), \Delta(n, -2m) \end{matrix}\right], \tag{4.2}$$

where $R(\varrho + 2s + 2ms \pm \nu \pm 2ms - \lambda) > 0,\ 2s > n$

$|\arg a^{2s}| < (2s - n)\frac{\pi}{2}$ and $R(p) > 0$.

COROLLARY. On taking $\lambda = -\frac{3}{2}$ and letting $n \to 2,\ s \to 1$ we see that if

$$\phi(p) \underset{\nu}{\overset{k}{=\!=}} t^{\varrho} f(t)$$

and

$$\psi(p) \underset{k,\, m}{\overset{v}{=\!=}} f\left(\frac{1}{t}\right),$$

then

$$\phi(p) = p^{1-\varrho}\, 2^{\varrho-k-m-\frac{1}{2}}$$

$$\times \int_0^\infty \psi(t).\, G^{2,\, 2}_{2,\, 6}\left[\frac{p^2 t^2}{16} \middle| \begin{matrix} \dfrac{2k-2m-1}{4}, \dfrac{2k-2m-3}{4} \\ \dfrac{1+2\varrho+2\nu}{4}, \dfrac{1+2\varrho-2\nu}{4}, -\frac{1}{2}, 0, -m-\frac{1}{2}, -m \end{matrix}\right] dt \tag{4.3}$$

provided that the integral is convergent, the MEIJER transform of $|t^{\varrho} f(t)|$ and the VARMA transform of $|f(\frac{1}{t})|$ exist, and $p$ is real and positive.

The case in which $k + m = \frac{1}{2}$ in (4.3) has been given by SAXENA [6, p. 166].

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